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The Focusing Effect of Technology: Implications for Teacher Education

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When technology is implemented in classrooms, students often form ideas that are unexpected and unwanted by the teachers and the designers of the technology. This article advances the notion of the *focusing effect of technology* as a way of systematically accounting for the role of technology in such situations. A focusing effect refers to the direction of students' attention toward certain properties of the subject matter domain over others, brought about by the use of particular technological tools. Technology focuses students' attention in ways that are often not anticipated in advance and can have unintended consequences for students' conceptions. Vignettes are presented from two research studies, one involving graphing calculators and one involving mathematics software called *SimCalc Mathworlds*. The research findings are synthesized and reinterpreted in order to illustrate and develop the notion of the focusing effect of technology. The significance of this construct lies in the connection that it affords between individual students' conceptions and the way technology is used in the instructional environment. Implications for teaching and for the preparation of teachers in the use of technology are discussed.

A number of researchers have critiqued the idea that computer software or devices such as graphing calculators can embody, in a transparent manner, the subject matter concepts that designers so carefully try to embed in the technology (Holyes & Noss, 1992; Lesh, Post, & Behr, 1987; Meira, 1998; Teasely & Roschelle, 1993). In fact, students routinely form ideas and interpret technological environments in ways that vary widely from what teachers and designers expect. Researchers relying upon a constructivist perspective explain why students "see" technology differently from adults by pointing to the role of prior knowledge and experiences (Bowers & Nickerson, 2000; Kieren, 1994; Noble, Nemirovsky, Tierney, & Wright, 1999). In an oft-cited example, Goldenberg (1988) illustrated how students view representations of functions in a graphing software environment differently from mathematically sophisticated adults. If one is familiar with graphs of linear functions, then he or she will expect the graph of the line $y = -5x - 1$ to move up vertically when the y-intercept is changed from -1 to +1. However, beginners lack such algebraic sophistication and are more likely to perceive that the line segment moves diagonally from the bottom left corner of the computer screen to the top right corner.

This article argues that inherent features of technology also play a critical role in situations in which students form unintended ideas. The notion of the *focusing effect of technology* is advanced in this article. A focusing effect refers to the regular direction of users' attention toward certain properties or patterns in the subject matter domain, which is brought about by the use of particular technological tools. Technology can focus students' attention in ways that are often not anticipated in advance but which may have unintended consequences for students' conceptions. The construct of focusing effects emerged from a synthesis of a series of studies conducted as part of an ongoing research project, the Generalization of Learning Mathematics in Multimedia Environments.

The purpose of this article is to characterize the focusing effect of technology, identify the theoretical roots of this notion, illustrate the focusing effect of technology with two examples from research studies, and explicate the significance of this notion for teaching and teacher education. The examples presented here are drawn from two larger studies: (a) an investigation of a high school classroom implementation of a reform mathematics curricula in which graphing calculators were used regularly; and (b) a teaching experiment conducted with secondary school students using simulation software called *Mathworlds* (1996), which was developed for the SimCalc Project (directed by James Kaput). In both cases, a major mathematical goal of the instruction was to help students construct ratios and rates as appropriate measures in real world situations. Although the content domain of the examples is mathematics and the technology involves graphing calculators and computer software, the phenomena of focusing effects is applicable to other content domains and to other types of technology.

CHARACTERIZATION OF THE FOCUSING EFFECT OF TECHNOLOGY

The focusing effect of technology refers to the regular direction of users' attention toward certain subject matter properties over others. This focus is brought about by the set of the affordances and constraints of the technological environment. Every tool affords or supports particular types of activity or actions (Gibson, 1986). For example, the speed simulation environment in *Mathworlds*, which will be described in detail later in the article, affords the activity of determining whether or not two characters walk at the same speed by visually inspecting whether they travel "neck-and-neck" for the duration of the shorter journey. A constraint is a regularity that holds in some domain and typically limits activity (Barwise & Perry, 1983). A constraint built into *Mathworlds* is that "same speed" must be determined by the user; a student cannot ask the computer to tell him or her whether the characters are walking at the same speed. Affordances and constraints are not simply features of the technology but rather are relationships between resources in the environment and users of the environment (Gibson, 1986). Furthermore, focusing effects emerge in specific instructional environments and thus interact with many factors such as the instructor's behavior, shared mathematical language, students' knowledge and goals, and features of the curricular materials.

The construct of focusing effects is rooted theoretically in a view of abstraction in context. Abstraction involves the identification of regularities in one's activities, the isolation of certain properties, and the suppression of other details (Frorer, Hazzan, & Manes, 1997; Harel & Tall, 1991). For example, a young child constructs "sixness" by identifying a property common to her actions on sets of six spoons, six bowls, and six plates (namely, cardinality) while ignoring other properties (e.g., color) and then considering the common feature in isolation. Hershkowitz, Schwarz, and Dreyfus (2001) pointed to the importance of attending to the multifaceted context in which the process of abstraction occurs: "A process of abstraction is influenced by the task(s) on which students work; it may capitalize on tools and other artifacts; it depends on the personal histories of students and teachers; and it takes place in a particular social and physical setting" (p. 196). The notion of focusing effects accounts for the role of technological artifacts in the activity of directing students' attention toward particular aspects of mathematical activity.

Abstraction is one of the processes involved in the creation of mental structures (von Glasersfeld, 1995). Thus, there is a relationship between students' attention to particular mathematical regularities and the ideas or conceptions that they form. Abstraction is also related to generalization, the extension of an existing mental structure to new objects (Harel & Tall, 1991). Consider the well-documented example of children's often inappropriate extension of the concept "multiplication makes bigger" from the domain of whole numbers to rational numbers (Greer, 1992). Years of experience with whole numbers likely direct students' attention to the pattern that the product is always larger than either factor when both factors are whole numbers. Thus, students generalize on the basis of that experience. This article investigates the ways in which features of technological environments support particular conceptions and generalizations by directing students' attention toward specific mathematical regularities and properties over others. In short, the focusing effects of technology can be seen as one mediating factor between the instructional environment and individual students' conceptions.

COMPUTER SOFTWARE EXAMPLE: *SIMCALC MATHWORLDS*

The first vignette is drawn from a teaching experiment conducted in a university computer lab for about 30 hours during the summer of 1999. Nine average-performing students (i.e., those who earned Bs or Cs) were recruited from 8th-10th-grade math classes. The first author of this article was the instructor for the teaching experiment (for details of the study, see Lobato & Thanheiser, 2000, 2002).

Students engaged in a "same speed" activity using *Mathworlds* (Figure 1). An animated clown walked at a constant speed across the computer screen covering 10 cm in 4 sec. Students were asked to enter time and distance values for a second animated character, a frog, so that the frog would walk at the same speed as the clown. The *Mathworlds* software was used because of its potential to help students develop an understanding of a ratio as a measure of speed. The software simulates constant speed, which is very difficult to attain in one's everyday experience of walking, and it allows students to make and test conjectures regarding the effects of varying distance and time on speed.

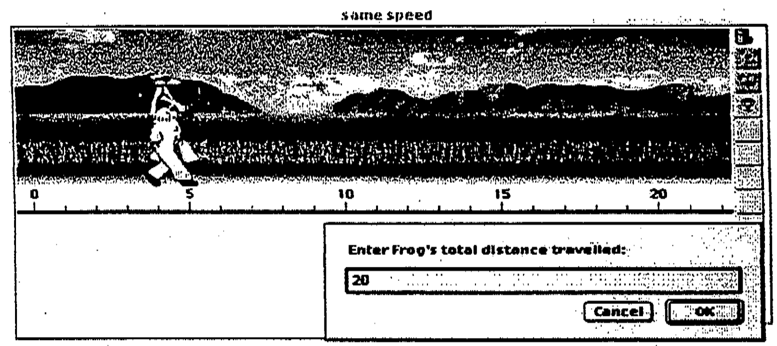

Figure1. Screen capture from *SimCalc Mathworlds*

The "same speed" activity was unexpectedly challenging for all of the students, and an unintended focusing effect emerged. Many students relied upon guess-and-check strategies to generate "same speed" values. For example, one student tried 15 cm and 8 sec and then kept adjusting the time values until he correctly arrived at 15 cm in 6 sec. Other students identified numeric patterns such as multiplying the distance and the time by the same whole number. However, two major limitations to the students' work soon became evident. First, when the researcher questioned students, no one was able to explain *why* their numeric patterns worked.

Second, students' illustrations indicated a lack of connection between their numeric strategies and the proportional relationship between distance and time in the speed situation. During the class discussion, students were asked to explain why walking 20 cm in 8 sec was the same speed as walking 10 cm in 4 sec. Terry used a numeric doubling pattern, namely that 20 is 10 x 2 and that 8 is 4 x 2. However, he could not explain why this pattern worked, and his drawing did not represent the doubling pattern. He drew two line segments, one representing 10 cm for the clown and one representing 20 cm for the frog (Figure 2). The frog's segment was only slightly longer than the clown's segment, and nowhere near double the length of the 10-cm segment. Terry explained that for both characters to have the same speed they would need to walk 10 cm in 4 sec at the same time. Terry's explanation fits the students' experience of determining whether or not two characters walked at the same speed during the computer simulation by watching to see if the characters walked "neck-and-neck" during the first 10 cm and ignoring the rest of the frog's journey. This might explain why Terry did not account for the remainder of the frog's journey and why the numeric strategy of multiplying the distance and time by 2 was not represented in his drawing. The next student, Jim, offered a similar drawing and numeric strategy, and the discussion appeared to stall.

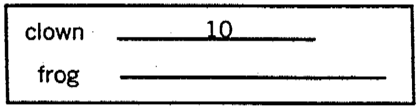

Figure 2. Authors' recreation of Terry's nonproportional drawing of 10 cm and 20 cm

The focusing effect in this example involved a direction toward the first 10-cm segment and away from what happened in the rest of the second character's journey. In order to check a potential "same speed" value by running the computer simulation, students developed a practice of checking to see whether the characters walked at the same speed by looking to see if the characters walked "neck-and-neck" for the duration of the shorter journey. This affordance allowed students to experiment with speed and create a set of correct "same speed" values. However, to develop more sophisticated reasoning involving the formation of an equivalent set of ratios as a measure of speed, it is critical to focus students' attention on the proportionality of the frog's entire journey.

GRAPHING CALCULATOR EXAMPLE

The second vignette originated from a study examining students' generalization of key linear functions concepts in an Algebra 1 classroom that regularly used graphing calculators (Lobato, Ellis, & Muñoz, in press). The class used the *Contemporary Mathematics in Context* materials produced by the Core-Plus Mathematics Project (Coxford, Fey, Hirsch, Schoen, Burrill, Hart, Watkins, Messenger, & Ritsema, 1998). Interviews with seven students revealed an interpretation of the slope of a linear equation previously unreported in the literature: Over half of the subjects identified the slope value as the scale of the x-axis. Rather than understanding m in $y = b + mx$ as a ratio of the change in y-values to the change in corresponding x-values, the students interpreted m as "what it goes up by," which appeared to mean "what the x's go up by" along the x-axis. For example, one student wrote the equation $y = 5 + 2x$ given graphical data in which the scale of the x-axis was 2. She justified her choice by explaining, "This is the y-axis and this is the x-axis ... so it's going up by 5, the y, and the x is going up by 2."

The interpretation of m as the scale of the x-axis was unexpected, particularly because the nature of a linear function is not dependent upon the value chosen for the scale of either axis. As a result, classroom data were analyzed to determine how the instructional environment supported this conception. The analysis revealed that the use of the graphing calculator played a key role. The following discussion will show how the graphing calculator contributed to an inappropriate conflation of the change in x-values of a function with the scale of the x-axis. (See Lobato, Ellis, & Muñoz, in press, for an account of how additional classroom features further led to an association of both of these constructs with the slope of a linear equation.)

The students created both graphs and tables of linear data with the graphing calculator. To create a table of values for a function, students first entered an equation and then used the table setup feature shown in Figure 3. To use this feature, students must determine the table's initial x-value, called TblStart, and the change in x-values, called Δ Tbl. In a typical activity, the teacher asked students to enter the equation $y = 2.7x$ and then use the table setup feature to find the value of y when $x = 2.5$. Students experimented with different values for TblStart and Δ Tbl until they produced a table including the x-value of 2.5. Since the students had to articulate a value for Δ Tbl every time they created a new table, their attention was likely drawn to the iterative change in x-values in this situation rather than to a relationship between corresponding x- and y-values. Additionally, the teacher used the phrase "going by" to refer to Δ Tbl: "What could we put on table setup to get that to come out ... you could start at 0 [referring to the TblStart input] going by point fives [referring to the Δ Tbl input] if you guys want to." As shown below, the "going by" language helped create an inappropriate link between the change in x-values and the scale of the x-axis.

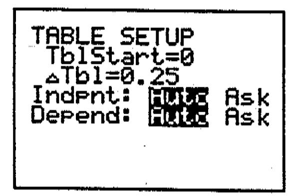

Figure 3. TI-83 commands for creating a table

The only other nonarithmetic calculator function used in this instructional unit was the Window function, which allows users to set the parameters for graphs (Figure 4). After the students entered data or an equation into their calculators, they used the Window function to input the minimum and maximum values for x, y, and the scale of each axis. The teacher and students referred to the value for Xscl (the scale of the x-axis) as "what the x's go up by." The same language was used to refer to Δ Tbl (the change in x-values), which likely helped link the two constructs for students. Furthermore, the graphing calculator requires the user to input a value for the scale of the x-axis before viewing a graph or using a table. The teacher's attention to this requirement highlighted the importance of considering the scales of the axes: she required that the students articulate a value for the scale of the x-axis for every calculator-related problem. Thus, the instructional environment emphasized a focus on the scale of the x-axis to a degree that might not have occurred without the use of the calculator.

```
WINDOW
 Xmin=0
 Xmax=6
 Xscl=1
 Ymin=0
 Ymax=12
 Yscl=1
 Xres=█
```

Figure 4. TI-83 commands for creating a graph

Students in the classroom demonstrated a conflation of the change in x-values with the scale of the x-axis. For example, during a small group activity, students used their graphing calculators to create a scatterplot given a table of linear data. Enrique helped Marina determine what to enter for each calculator prompt in the Window function (Figure 4). Enrique, looking at the Xscl prompt on the calculator, asked Marina, "What are we going up by?" Marina responded, "It's going by 5, 6, 7, 7.5, oh, it's going up 2.5," which was the change in x-values in the table. While Enrique used "going up by" to refer to the scale of the x-axis, Marina interpreted his question in terms of the change in x-values in the table. This type of exchange occurred repeatedly throughout the instructional unit, indicating that some students did not distinguish between the two constructs.

The focusing effects of the graphing calculator supported an inappropriate association between the scale of the x-axis and the change in x-values of the function. The ambiguity of the "goes up by" language linked the two construct since the phrase "what the x's go up by" was used to simultaneously refer to ΔTbl and Xscl. Other focusing effects included the following:

(a) the commands ΔTbl and Xscl were mediated by the same physical device; (b) the graphing calculator requires the user to input a value for ΔTbl before viewing a table, thus likely drawing more attention to the iterative change in x-values than to the functional relationship between x- and y-values; (c) the graphing calculator requires the user to input a value for Xscl before viewing a graph, thus drawing attention to the scale of the x-axis scale; and (d) the table set and window functions were the only nonarithmetic calculator functions used in the instructional unit. Finally, when creating a graph from a table, the teacher always used the change in x-values as the parameter for Xscl, further linking the two constructs. The students may not have understood that the x-values of a function are independent of the scale of the x-axis, and that the ΔTbl and Xscl values do not have to be the same.

PEDAGOGICAL PRINCIPLES FOR ADDRESSING FOCUSING EFFECTS

In both vignettes, the technology unexpectedly drew attention to some feature of the situation that led to unintended mathematical conceptions or strategies. One way to address the focusing effects of technology is to redesign the technology to better align the affordances and constraints of the environment with the hypothesized mental operations of students (for an example of a program that appears to accomplish this, see Olive, 1999, 2000; Steffe & Olive, 1996). However, teachers rarely have the power to change the technological environment. Thus, a more practical solution involves refocusing students' attention either by supplementing with "offscreen" activities (as we will illustrate in the conclusion to the *Mathworlds* example) or by using the technology differently (as will be illustrated with a graphing calculator example).

Four pedagogical principles can be applied to address unintended focusing effects of technology:

1. Identify the focusing effects of technology by carefully examining what students attend to as they interact with the technology. It is not usually possible to identify focusing effects in advance because they are usually unintended and because they interact with features of the instructional environment.
2. Identify a new object of focus. Before the focusing effects can be altered, teachers need to identify the desired object of focus in the particular subject matter domain. This often involves a shift from calculational to conceptual goals.

3. Problematize the object of focus. Students need to encounter problem situations that necessitate or problematize the mathematical object of focus. For example, in the *Mathworlds* example, students were able to negotiate the tasks by simply varying one quantity at a time (by guessing and checking) rather than by forming a ratio of covarying quantities. They were also able to determine whether or not their guesses were correct by looking only at the duration of the shorter journey without considering the proportional nature of the relationship between distance and time in the situation. Problematizing requires the creation of tasks that present a need for students to develop the target knowledge.
4. Focus attention in an alternative direction. The final step is to create additional activities or use technology in ways that modify the focusing effect. The following specific instructional examples include the use of diagrams, explanations, alternative features of the technological environment, and teacher questions—all of which direct students' attention in an alternative direction.

MATHWORLDS REVISITED

The class discussion of the "same speed" activity shows how the focusing effect of a computer software program can be addressed. The pedagogical principles will be highlighted in the discussion. First, the teacher/researcher identified the students' difficulties, as noted in the *Mathworlds* vignette. This is significant because many of the students produced correct "same speed" values. When correct answers are produced, it is easy to overlook potential conceptual problems. However, the teacher recognized the significance of the students' inability to explain their numeric patterns or to connect their numeric strategies with the speed situation. Consequently, the teacher's goal for the discussion involved a different object of focus. Specifically, the teacher wanted students to construct an equivalence class of ratios as a measure of speed and to connect the numeric strategy of doubling to the speed situation. One way to create a ratio so that it is connected to the quantities in the situation is illustrated in the following argument: If one walks 10 cm in 4 sec and then 10 cm in 4 sec again, without speeding up or slowing down, then he or she will not have altered his or her speed; hence, walking 20 cm in 8 sec is the same speed as 10 cm in 4 sec. This reasoning indicates the formation of "10 cm in 4 sec" as a composite unit (Lamon, 1995), which allows the ratio 10:4 to be operated upon as a single entity and allows for the formation of the equivalent ratio 20:8 as two of the unit 10:4. The argument also preserves the constancy or invariance of speed.

The teacher addressed the focusing effect of the technology by drawing attention to the entire journey and consequently helped students construct ratios as composite units. She used three instructional techniques: (a) engaging in off-screen activities until the students provided evidence of attending to the new object of focus; (b) requiring that every student draw a picture of the situation on the board; (c) continuing the discussion until every student was able to explain why doubling and other numeric strategies worked. Specifically, after Terry and Jim created their nonproportional drawings, the teacher persisted until someone could explain why the numeric strategy of doubling worked. The insistence on explaining and drawing pictures served to problematize the construction of a ratio by creating a necessity to communicate and to formulate an explanation as opposed to the need to calculate, which is what the computer activity necessitated.

A breakthrough occurred when Brad constructed a ratio and explained why doubling works:

> Because the clown is walking the same distance; it's just that he's walking the distance twice...he's walking it once, going all the way here [Brad retraced Terry's line, up to 10 cm and drew a vertical mark]. Four seconds. OK. He's going to walk it again. Another 4 seconds, another 10 centimeters in four seconds [Brad extended Terry's line representing the frog].

Brad's picture illustrated what happened to the frog character after the initial 10 cm in 4 sec by noting that the frog walked another 10 cm in 4 sec. Brad's work was significant because it marked a turning point in the location of students' attention for the remainder of the whole class discussion. Subsequently, students began focusing on "10 cm in 4 sec" as a unit, which could be iterated and partitioned as a single entity. Students also began to account for the entire duration of the frog's journey. These foci differ significantly from the focus on changing one quantity at a time in the guess-and-check strategies employed in the computer activity and from the focus on only the duration of the shorter journey in the computer simulation.

Once the focus of attention had changed, students successfully formed and operated with ratios and connected those ratios to the quantities in the speed situation. For example, Denise added another 10 cm in 4 sec section onto Brad's drawing, concluding that 30 cm in 12 sec was also the same speed as 10 cm in 4 sec. Terry eventually produced drawings demonstrating proportional reasoning. For example, he explained why walking 2.5 cm in 1 sec was the same speed as walking 10 cm in 4 sec. Terry drew a segment representing the clown's trip of 10 cm in 4 sec. He then placed a vertical mark on the line segment representing about one-fourth of the length of the segment and wrote 2.5 and 1. He circled the 2.5 cm in 1 sec section (see Figure 5) and repeated the new 2.5:1 ratio three times, marking 2.5 and 1 on the drawing each time, until he recreated the original 10 cm in 4 sec segment (Figure 5). Terry demonstrated an understanding of an important proportional-reasoning idea when he stated that "it would be like he's [meaning the clown] walking one fourth of the 10 and 4; it's like one fourth of each thing." This suggests an understanding that traveling one fourth of the "10 cm in 4 sec" journey represents the same speed as walking "one fourth of 10" cm in "one fourth of 4" seconds. This illustrated a striking change in Terry's focus of attention. He accounted for the entire duration of each character's journey, composed distance and time into a unit and operated on that unit, and connected his numeric strategy with the associated meaning in the speed situation.

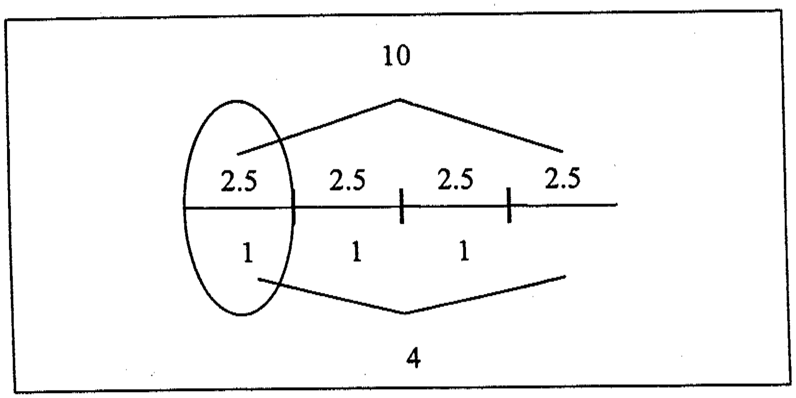

Figure 5. Authors' recreation of Terry's diagram showing why walking 2.5 cm in 1 sec is the same speed as walking 10 cm in 4 sec .

GRAPHING CALCULATOR REVISITED

The manner in which the students and the instructor used the graphing calculator helped focus students' attention on both the scale of the x-axis and on the change in x-values in a table of data. Many students failed to distinguish between these two constructs, not recognizing that the x-axis scale is not connected to the values of a function in a tabular form, or to its slope. Given the recognition that the graphing calculator's table setup function and window function contributed to a focus unintended by the instructor, a different instructional approach might have helped direct students' attention to a different object of focus.

The instructional object of focus needs to change from articulating values for the x-axis scale and the change in x-values to identifying that the nature of a linear function is not related to the scale of the x-axis. In the instructional environment described in this article, the teacher encouraged the students to create tables with the graphing calculator to identify y-values given particular x-values. The students created graphs on the calculator mainly to reproduce data already presented in graphical or tabular form.

One could problematize the distinction between the nature of a linear function and the scale of the x-axis by exploring how to determine y-values for given x-values from a table and a graph *simultaneously*. For example, the teacher and the students first used the graphing calculator in the instructional unit to create a table for the linear equation $y = 2.7x$. In an alternative treatment, the teacher could allow for the simultaneous examination of the graphical and tabular presentations by changing the mode of display to "G-T," which stands for "graph-table." This allows students to view the graph of the equation and a table of values side-by-side, as shown in Figure 6.

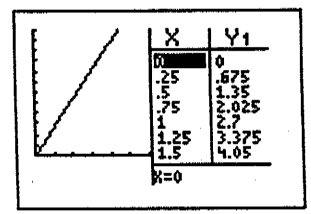

Figure 6. The graph-table mode of the TI-83

To use the G-T mode, students must still determine the scale of the axes for the graph, and the starting value and ΔTbl for the table. The instructor could encourage a focus on the distinction between Xscl and ΔTbl by entering different values in each case. For example, a reasonable value for the scale of the x-axis is 1. The teacher could then problematize the generation of a different value for ΔTbl by asking students to determine the corresponding y-values for a series of x-values, such as $x = 0$, $x = 1.5$, and $x = 2.75$. In this case, a value of 1 for ΔTbl would not allow students to determine y-values for $x = 1.5$ and 2.75. Students would instead have to choose a value such as 0.25 for ΔTbl. Thus students would be able to view the graph and the table together, identifying the x-axis scale as 1 and the change in x-values as 0.25. The teacher could direct students to notice that the change in

x-values, as well as the scale of the x-axis, can be anything they choose and are not part of the nature of the function.

The teacher could further focus attention on this difference by showing students how to use the function-tracing feature of the G-T mode to determine values directly from a graph (Figure 7). Students select "CALC" and then choose "value," which allows the user to pick any x-value. If one types in 1.5, the calculator places a cursor at $x = 1.5$ on the graph and displays the point (1.5, 4.05). The calculator automatically adjusts the table values to make the first entry in the table as close to $x = 1.5$ as possible (due to a programming feature of the calculator, the table value reflects the exact location of the cursor to a width of one pixel). This automatic change in the table, including the change in ΔTbl to the width of one pixel, again highlights the difference between the scale of the x-axis, which has not changed, and the change in x-values, which has. The instructor could explicitly direct students' attention to this difference by asking questions such as, "Where is the 1.5 on the graph? Where is the 4.05? Where are these values in the table?" In addition, problems that necessitate an exploration of the relationship between x- and y-values for many different points could eventually lead to the comparison that y is always 2.7 times as big as x for any corresponding x- and y-values.

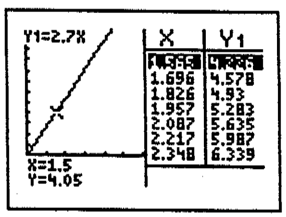

Figure 7. The function-tracing feature in the G-T mode of the TI-83

The alternate instructional suggestions previously outlined are based on a new object of focus, namely differentiating between the change in x-values and the scale of the x-axis. This focus could be problematized in part by requiring students to identify corresponding y-values for x-values that are not multiples of the number chosen for the scale of the x-axis. Finally, the alternative focus should be explicitly supported by the instructor through his or her questions, choice of tasks, and language. The split-screen function, in contrast to viewing tables and graphs separately, might better afford the recognition that the scale of the x-axis is not related to the nature of the function.

DISCUSSION AND IMPLICATIONS FOR TEACHER EDUCATION

This article demonstrated how the mathematical conceptions formed by students were related to the focusing effects of technology. Consequently by addressing and altering the focusing effects, teachers can significantly affect the nature of students' ideas. Thus, the construct of the focusing effect of technology has implications for teaching and for the preparation of teachers to use technology. Three implications for teacher education are briefly considered.

First, it is important to expose teachers to cases of students' thinking in technologically intensive environments, such as the two vignettes presented in this article. Teachers typically select software or other technology for use in the classroom because they believe the technology will clarify or illustrate certain concepts. Consequently, even though teachers may be cognizant that students may form undesirable interpretations of the technological environment, the actual occurrence of such in the classroom often generates a reaction of surprise. By examining cases of the specific ideas that students typically form as a result of interacting with technology, teachers are more likely to understand the interpretive nature of students' cognitive activity and the roles that the technology, the students, and the teachers play in that activity.

Second, teacher education courses should help prospective teachers recognize that students' unintended interpretations are reasonable given the combination of the focusing effects of technology, the particular instructional practices, and the students' goals and prior knowledge. In particular, teachers should be aware that they need to identify what students attend to when they interact with technology. Identifying the focusing effects of technology requires careful examination and time to listen to students. It is not usually possible to identify focusing effects prior to observing students' use of the technology because many focusing effects are unintended and occur in interaction with features of the instructional environment. Thus, it is important for teachers to position themselves as learners in the classroom and investigate the nature of students' work with the technology, rather than assume ahead of time what students will learn from their interactions with the technology.

Finally, the pedagogical principles developed in this article can be introduced in teacher education courses and can provide teachers with strategies for addressing focusing effects that contribute to unproductive or limited student conceptions. These principles include identifying a new object of focus, problematizing the object of focus, and focusing attention in an alternative direction. In particular, teachers should be encouraged to supplement technological experiences with "offscreen" activities designed to address the focusing effects of technology. The *Mathworlds* example illustrates the productive interplay between computer activities and offscreen activities that emphasize explanations and student-constructed representations.

It is important not to view focusing effects as negative. Although *Mathworlds* contributed an unintended focusing effect of directing attention toward the journey of shorter duration and away from what happened in the rest of the second character's journey, *Mathworlds* also played an important role in the classroom. The same affordance that contributed to problems for students also allowed students to experiment with speed and create a set of correct "same speed" values. Thus, focusing effects may create opportunities at the very same time that they contribute to the formation of limited or problematic conceptions. As a result, teachers should develop a cycle of working with technology, identifying limitations brought about by unintended focusing effects, and addressing those effects through supplementary off-screen activities or the use of technology in a different way. These new activities can lead to other focusing effects, and the iterative cycle can then be repeated.

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