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The Teacher's Role in Supporting Students' Connections Between Realistic Situations and Conventional Symbol Systems

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We use the notion of *focusing phenomena* to help explain how a teacher's actions were connected to her students' interpretations of a linear equation. This study was conducted in a high-school classroom that regularly emphasised dependency relationships in real-world situations. Seven interviews revealed a majority view of $y = b + mx$ as a *storage container*—a place to insert b and m values—rather than as a relationship between x- and y-values. Classroom analysis revealed how the teacher directed attention away from functional relationships with increasing frequency as she moved from realistic situations to conventional representations.

Current reform efforts in algebra instruction in several countries emphasise mathematical modelling, the use of realistic problem situations, and a pedagogical approach toward functions based on the notions of change and dependency (Fey, 1990; Kaput & Nemirovsky, 1995; Kieran, 1993). According to the Australian Education Council (1990), the study of functions should begin with modelling activities that draw upon students' daily experiences with variation. Similarly, algebra standards in the USA call for middle- and high-school students to identify functions modeling quantitative relationships and to analyse rates of changes in various contexts (National Council of Teachers of Mathematics, 1989, 2000).

One rationale for the use of real-world situations in algebra is their potential to lend meaning to the conventional symbol systems of tables, equations, and graphs by connecting them with students' informal understanding of dependency relationships (Mathematical Sciences Education Board, 1998; National Research Council, 1990). Our challenge is to support the transition to using conventional symbols in a manner that allows the symbols to carry the meaning of acting on experientially real quantities (Gravemeijer, Cobb, Bowers, & Whitenack, 2000). An emerging literature documents students' learning trajectories as they explore the conventional symbol systems for functions linked with experientially real settings (Bowers & Nickerson, 2000; Kaput, 1994; Nemirovsky, Tierney, & Wright, 1998). While these studies focus on students' conceptions and difficulties, further research is needed to understand the role of teachers in supporting the movement between exploring dependency relationships in realistic situations and using conventional symbol systems to represent those situations.

The purpose of this study is to examine how the teacher's instructional actions are connected to students' interpretations of the meaning of a linear equation. The study was conducted in a secondary classroom in the USA using the *Contemporary Mathematics in Context* materials produced by the Core-Plus Mathematics Project, or CPMP (Coxford et al., 1998). We chose CPMP for two reasons. First, the materials regularly explore functions as a way to describe dependency relationships in complex real-world situations. Second, they emphasise connections among the conventional symbol systems of tables, equations, and graphs. We investigated linear functions because it is a conceptually complex topic for students, rich in terms of real-world connections, and featured as an important topic in CPMP and in the reform standards set forth by the National Council of Teachers of Mathematics (1989, 2000).

Conceptual Framework

Researchers operating from the Realistic Mathematics Education approach (developed at the Freudenthal Institute) use the notion of *progressive formalisation* to model how students move from informal reasoning in realistic situations to the use of formal conventional symbol systems (Gravemeijer et al., 2000; Streefland, 1995; van Reeuwijk & Wijers, 1997). Van Reeuwijk (2002) summarises the general approach as it applies to algebraic reasoning: (a) observe patterns or regularities among quantities in experientially real situations; (b) describe chains of calculations using nonconventional symbols such as arrows, arithmetic operations, and arithmetic trees; (c) generalise nonconventional symbols to informal word formulas; (d) create formulas and use different representations (tables, direct and recursive formulas) to describe more complex situations; (e) create, interpret, and test algebraic formulas and conventional rules; and (f) derive formulas from other representations, such as tables. Other researchers describe a similar process even when students work with a number pattern or an arithmetic object not necessarily linked to realistic contexts: identify a pattern; test to see if the pattern generalises; express generalisable patterns using informal verbal and written statements; and develop algebraic notation as a concise way to describe generalised patterns (Pegg & Redden, 1990; Reid, 2002).

Although exemplars of this type of progression exist empirically (see, e.g., van Reeuwijk & Wijers, 1997), recent research indicates that the process can be far from straightforward. For example, Lee (1996) found that while students readily notice patterns, they often find patterns that do not extend widely to different cases and are not "algebraically useful" (p. 95). Another common finding is that students tend to focus on iterative or recursive patterns of differences between successive terms in data rather than on functional patterns (Stacey, 1989; Stacey & MacGregor, 1997; Szombathelyi & Szarvas, 1998). Similarly, students often treat the growth patterns in each column of a table as separate and uncoordinated (Mason, 1996; Schliemann, Carraher, & Brizuela, 2001). Even when students do attend to useful patterns, the perception of a valid number pattern does not guarantee the ability to correctly generalise and formalise that pattern (English & Warren, 1995; Orton & Orton, 1994).

Competencies as well as difficulties have been identified for students who move from informal settings to conventional representations, however, less is known about the teacher's role in facilitating this development. In this paper, we extend the notion of *focusing phenomena*, as advanced by Lobato, Ellis, and Muñoz (in press), to help explain how the teacher's instructional actions are connected to her students' interpretations of the meaning of a linear equation.

Focusing phenomena are observable features of the classroom environment that regularly direct attention to certain mathematical properties or patterns. Focusing phenomena emerge not only through the instructor's behavior but also through co-constructed mathematical language, features of the curricular materials, and the use of artifacts such as graphing calculators. The construct of focusing phenomena is rooted in a situated view of the abstracting process. Abstracting involves the identification of regularities in one's activities, the isolation of certain properties, and the suppression of other details (Frorer, Hazzan, & Manes, 1997). Hershkowitz, Schwarz, and Dreyfus (2001) situate the abstraction process by arguing that abstracting is influenced by the tasks on which students work, the artifacts and tools available, the personal histories of students and teachers, and the particular social and physical setting. Thus, the notion of focusing phenomena accounts for multiple agents that combine to direct students' attention to particular aspects of mathematical activity. We examine how a teacher directs her students' attention toward or away from relationships between independent and dependent variables depending on whether they discuss a realistic situation or a conventional representation of the situation. By developing a greater understanding of the teacher's role in helping students connect realistic situations with conventional symbols, we hope to gain insight into how to design productive instructional interventions.

Research Methods

Participants and Data Collection

This study emerged out of a continuing 5-year research project, the *Generalisation of Learning Mathematics Project*. Data collection occurred in a large urban high school in the southwestern USA. Seventy-five percent of the school's 2,400 students are Hispanic, and the remaining students are Filipino (13.5%), Caucasian (5%), African American (3.5%), Asian or Pacific Islander (2.5%), and Native American (0.5%). A quarter of the student population is classified as limited English-proficient, and the students perform below the nation's average on standardised achievement tests for mathematics and reading. The school was selected because it was one of only two schools in the metropolitan area using the CPMP materials, and because no other local high schools used materials that emphasised the grounding of algebraic reasoning in realistic situations as regularly as CPMP.

The collaborating teacher, called Ms R for the purposes of this study, had taught for 6 years at the time of the study. She was recruited in part because she fitted the criteria set by Huntley, Rasmussen, Villarubi, Sangton, and Fey (2000) for appropriate implementation of CPMP; that is, she followed the intended curriculum, used graphing calculators in the classroom, and encouraged cooperative learning strategies in heterogeneous groups. Ms R was enthusiastic about using real-world situations because she wanted her students to make connections between mathematics and their daily lives. Data collection occurred in the Course 1 class ($n = 36$) that Ms R deemed the most productive in terms of classroom management and students' readiness to learn.

Both interview data and videotaped classroom data were collected. Seven students participated in one semi-structured interview (Bernard, 1988). The interview occurred approximately four weeks into the instructional unit, after the relevant topic of linear equations in the $y = b + mx$ form had been developed. The interview lasted about 45 minutes and was videotaped. The interview sample was theoretically relevant rather than random. A significant number of the class members appeared to be unprepared for the CPMP program due to their limited background knowledge. Since the intent of the study was to examine students' interpretations of linear equations that developed out of investigations with real-world situations, it was important to select students who appeared to be capable of making sense of the CPMP materials. Specifically, interview participants were chosen based on Ms R's identification of students who earned high mathematics grades (in the A-C range), were willing to participate in classroom discussions, completed their homework assignments, and generally appeared to be well prepared for the CPMP curriculum. The sample included three of the four top-performing students in the class. Gender-preserving pseudonyms have been used for all participants.

Videotaped classroom data were collected for 5 weeks. One camera focused on the teacher during whole-class discussions and another focused on a target group of four students during periods of group work. The class met three times a week, once for 60 minutes and twice for 90 minutes. Two or three researchers observed each class session. One operated the video camera, and the other researcher(s) took detailed field notes (Schatzman & Strauss, 1973). Great effort was made to establish rapport with all students and particularly with the interview participants. Classroom observations occurred for two weeks prior to collecting videotaped data, so the participants were familiar with the researchers by the time the interviews occurred. As evidence of this familiarity, all of the participants freely responded to some interview questions by stating that they did not know the answer. Thus the researchers attributed a reasonable degree of validity to the responses that the students did provide. Every effort was made to minimise responses based on social pressure.

Overview of the CPMP Instructional Unit

The classroom observations occurred during 5 weeks of instruction on linear models. Throughout this paper, we use "instructional unit" to refer to the portion that we observed, even though it was about two-thirds of the textbook chapter. The remainder of the chapter addressed systems of linear equations. The instructional unit was organised around understanding linear equations in the slope-intercept form and making connections among tables, equations, and graphs. The unit consisted of several multi-day lessons in which ideas about linearity were developed through investigating real-world situations. Figure 1 provides an overview of the mathematical ideas and the realistic situations as they were explored in the classroom.

The CPMP approach relies upon modelling and linked representations to develop algebraic ideas. For example, the unit began with a 3-day activity in which students explored a linear relationship between the distance of an overhead projector from a wall and the enlargement factor between lines and their projected images.

	Situations	Topics	Day	Mathematical Ideas
Week 1	Overhead Projectors		1	Collect data and make predictions
			2	Write equation and graph for data
	TV Ratings	Exploration of Linear Data	3	Explore relationships between two quantities
Week 2	Concert Attendance		4	Predict using tables and scatter plots
			5	Find a linear model for a scatterplot
			6	Use the calculator to locate a line of best fit
Week 3	Rubber Bands and Springs	Exploration of Rate of Change	7	Introduce rate of change as $\frac{\Delta y}{\Delta x}$
			8	Calculate and explore rate of change
Week 4	Cost For Soda Machines	Slope, y-intercept and $y = b + mx$ Formally Defined	9	Define slope as a constant rate of change
			10	Write equations in $y = b + mx$ form.
	Individual Interviews Conducted After Day 10 and Before Day 14			
	Equations in Graphical Settings		11	Compare $y = b + mx$ and recursive forms of equations (y = now + next)
Week 5		Connections Among Tables, Equations and Graphs	12	Graph $y = b + mx$ equations
			13	Explore the effect of changing b and m on graphs of equations
	Speed with Motion Detectors		14	Connect speed phenomena with equations and graphs

Figure 1. Overview of the development of linear equations in the CPMP Unit.

By collecting and representing data with tables, students were able to experience an approximately linear relationship between the distance of the projector from the wall and the enlargement factor. Students used graphing calculators to investigate whether or not a given equation represented a good linear model for a particular scatterplot, thus gaining informal experiences linking linear equations with graphs. Students also informally explored the meaning of the y-intercept and the rate of change of a variety of linear functions in real-world situations before the terms y-intercept and slope were introduced.

Lessons were designed to promote cooperative learning, and small-group activities occurred during 12 of the 15 class sessions. The textbook included many exploratory and discovery-oriented problems. The teacher used a variety of instructional formats, including whole class discussion, whole class data collection, small group activities, and some direct instruction. She faithfully followed the CPMP curriculum, except for supplementing with additional practice and with a lesson on motion detectors.

Interview Instrument

The interview involved a series of questions about linear function concepts related to a single context (see Figure 2). The boogie board data appeared in a CPMP practice problem (which students saw on Day 5 in class); however, the data were presented in the interview in graphical rather than tabular form. The questions were developed to identify the sense students made of linear data and equations from a familiar data set. They were not designed to evaluate the CPMP curriculum and therefore did not span the range of tasks found in the CPMP text. The boogie board graph was constructed so that the point of intersection of the coordinate axes was (76, 0) rather than (0, 0). As a result, it was possible to identify whether students understood the meaning of y-intercept as the point where the line crossed the y-axis (regardless of the value of x) or as the value of the function when $x = 0$. The students had seen two similar examples in class prior to the interview, and thus the non-standard graph was considered appropriate for the task. The results of students' responses to the following two questions, which were designed to investigate their understanding of a linear equation, are presented in this paper.

1. [After being asked to draw an approximate line of best fit for the data]: Can you write an equation or formula for the line you drew? What does this equation tell you about the situation? [After the student identified an m or b value]: What does that number mean? What does the x in your equation mean?

2. Someone else I interviewed drew a line just like yours and came up with an equation of $y = -112 + 1.6x$. Do you think this equation is correct or incorrect? Why? What does each number in the equation mean? What does the x mean?

The *Oh-So-Cool* Surf Company rents boogie boards at Long Beach. Naturally, their business is affected by the weather. The graph below shows boogie board rental data from nine weekend days during July and August 1999.

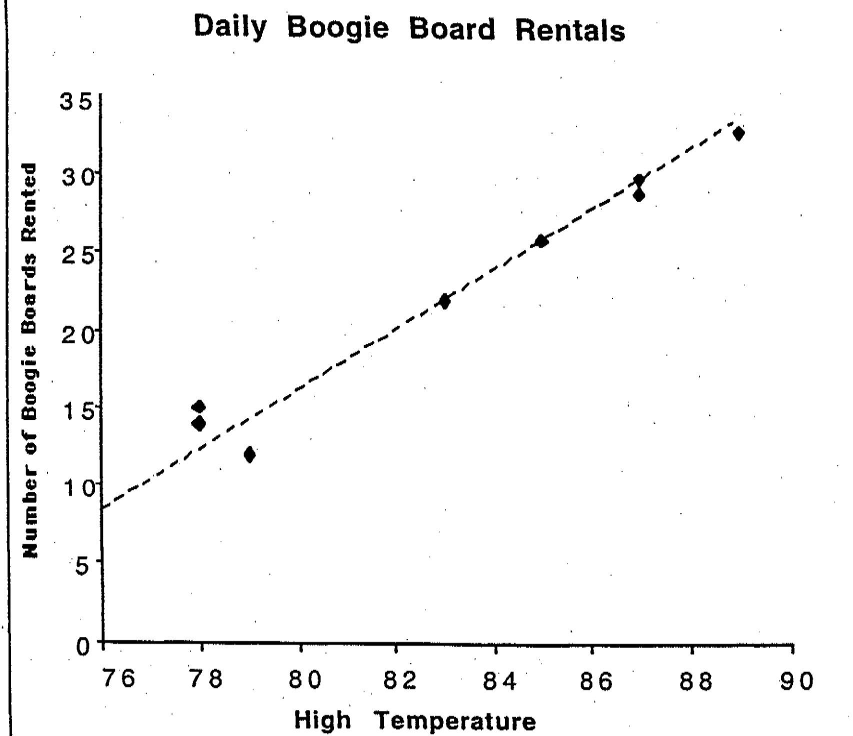

Figure 2. Situation presented in Interview 1.
(with one student's line of best fit indicated by the dotted line)

Results of the Interviews

In this section we provide evidence that the students appeared to interpret a linear equation of the form $y = b + mx$ as a *storage container*—a place to insert b and m values—rather than as a relationship between corresponding x- and y-values. First, we present students' interpretations of the meaning of linear equations, which emerged from the task of writing an equation for a line. Second, we present the students' interpretations for the meaning of x in the equation. These interpretations are closely linked to the students' understanding of the equation as a whole. Finally, we show how the notion of a linear equation as a storage container emerged from the analysis of the data.

Students' Interpretations of Linear Equations

Six of the 7 interview participants were able to write an equation for the given line (Figure 2). Each of these students wrote an equation in the form $y = \square \pm \square x$ and referred to the value in the first box as the "starting point" or "starting value" and the value of the second box as what "it goes up by". Thus, the students appeared to have generalised a linear equation as "y equals the 'starting point' plus 'what it goes up by' x." Enrique's work is typical:

Enrique: This is y, and then she put like a little x or whatever [writes $y = \square \pm \square x$].
Interviewer: Okay.
Enrique: And plus or minus and this [points to second box] and that [points to first box].
Interviewer: Okay. And what part do you remember about that [points to the first box]?
Enrique: This is like the starting value [points to the first box].
Interviewer: Okay.
Enrique: And this is like a slope or something like that [points to the second box].
Interviewer: A slope or something like that? What's a slope?
Enrique: The rate of change.
Interviewer: Okay.
Enrique: So that's what it's going up by.
Interviewer: Okay.

Students constructed various meanings for "starting point" and "goes up by". For example, Enrique wrote the equation $y = 76 + 2x$. He apparently interpreted the "starting point" as the initial value shown on the x-axis or "where the values on the x-axis start". The value 2 represented the scale of the x-axis or "what the x's go up by". In contrast, Carissa wrote the equation $y = 8 + 2.5x$. She interpreted the "starting point" as the point where the line starts, which is the location where the line intersects the y-axis regardless of the value of x. Carissa computed the 2.5 using the slope formula and referred to it as what the line "goes up by". The other meanings that students held for the two parameters are reported elsewhere (Lobato, Ellis, & Muñoz, in press). For the purposes of this paper, we will focus on the students' understanding of the equation as a whole, rather than on the students' meanings for each parameter.

The students either wrote the $y = \square \pm \square x$ template and then filled in values for the two boxes, or they wrote the equation from left to right, filling in values in the boxes as they proceeded. For example, Enrique wrote $y = \square \pm \square x$ and then placed 76 and 2 in the boxes. Another student, Carissa, proceeded from left to right. She wrote "$y = 8$" and explained that 8 was the starting point of the line. Then she wrote "+" and said that addition means the line will go up to the right. Finally, she calculated 2.5 using the slope formula and explained that 2.5 is "how much it's [meaning the line] going up by each time". She completed writing the equation as $y = 8 + 2.5x$. Carissa was then asked what the equation

tells her about the situation—a question that could elicit an understanding of the dependency relationship between the temperature and the number of boogie boards rented. Instead, she focused on values of the two boxes in $y = \square \pm \square x$: "That [pointing to the 8 in her equation $y = 8 + 2.5x$] um, is where they start off on, and they're adding 2.5 boogie boards." These examples illustrate how students treated the equation as a template for which one fills in missing "starting point" and "goes up by" values.

Meanings for x

Students were asked about the meaning of x for their produced equations and for the given equation $y = -112 + 1.6x$. This revealed information about the meaning that students generated for a linear equation as an entity. Our goal was to infer categories of meaning for the x value in the students' linear equations. Analysis of the transcriptions of the interview data followed the interpretive techniques in which the categories of meaning were induced from the data (Glaser & Strauss, 1967; Miles & Huberman, 1994; Strauss & Corbin, 1990). We present three categories of meaning for x.

x is a label. The most common meaning for x was demonstrated by 4 students, who interpreted x as a label meaning "goes by" or "every time". For example, when asked what x meant in $y = -112 + 1.6x$, Enrique responded that the x meant "every time" as indicated in the transcript below.

Interviewer: Okay. What does the x stand for?
Enrique: Like what is every time it goes up. Like 1.6x. Like every time.
Interviewer: So x means every time?
Enrique: Yeah.

Enrique pointed to the values along the x-axis, noting that "it's going up, yeah, because 88, 90 is about 1.6". Similarly, Linnea explained that the x in her equation $y = 5 + 2x$ meant "what the x-axis is going by". In Linnea's words, the $2x$ meant "the x-axis goes by 2s". Priyani reported that x meant "what it's going by". Raul stated that the x meant "every time" and the $2x$ in his equation meant "it goes up by 2 every time". In each case, x represented a label that was connected to the slope value.

x is part of a memorised equation. Two students reported that they did not know what the x meant in $y = \square \pm \square x$, but they knew that the x had to be there. Priyani was particularly articulate about this position: "In Ms. R's class, I just put x because it's y equal and it's [a] variable [equation], and you are supposed to put x, and I just put it, but I never know what it's for." Similarly, Carissa explained that "there's always an x" in a variable equation, but "I'm not sure why".

x represents a set of temperature values. Only the top-performing student in the class, Andy, provided evidence of an understanding that x represents a variety of temperature values that can be substituted into the equation in order to determine the corresponding number of boogie boards. Andy determined that $y = -112 + 1.6x$ was a reasonable equation by choosing a temperature (78), substituting 78 for x, calculating the corresponding value of y as 12.8, and checking the point (78, 12.8) against his graph, which was close to the actual point of (78, 14). When asked what the x means, he replied, "This is like you could fit any number, like 86, you could put in here it's just that you're like times-ing."

In striking contrast, none of the other participants appeared to understand when substitution was appropriate. For example, Carissa reported that she could substitute 31 boogie boards for x rather than y in $y = -112 + 1.6x$. Ruben mentioned that he should substitute a difference of two temperature values for x rather than a single temperature value. Priyani was unsure how to use her equation $y = 10 + 1x$ to determine the number of boogie boards rented when it was 80°. She wrote y = 10 + 80x but did not know how to proceed. The interviewer asked Priyani what the x and y represented on the graph, and she correctly responded that "x is the temperature and y is the number of boogie boards rented". Yet she was unable to use this information to solve the 80° prediction problem, because she did not know what to do with the 80 in her equation. Enrique was specifically asked if he could put numbers in for the x in $y = -112 + 1.6x$, and he said he did not know.

A Linear Equation as a Storage Container

The following factors suggest that the students, with the possible exception of Andy, viewed $y = \square \pm \square x$ as a storage container: (a) the prevalence of treating x as a label; (b) the lack of understanding that numerical values can be substituted for x to obtain corresponding y-values; and (c) the frequent verbalisation that an equation is "y equals the 'starting value' plus or minus 'what it goes up by' x". They appeared to treat the linear function as a container in which "starting point" and "goes up by" values were stored rather than as a relationship between corresponding x- and y-values. Furthermore, the data suggest that two students viewed the equation as two separate "equations" or statements rather than as a single entity. Specifically, these students appear to have read the equation $y = b + mx$ as "y 'is' or 'goes up by' b and x 'goes up by' m" (see Figure 3).

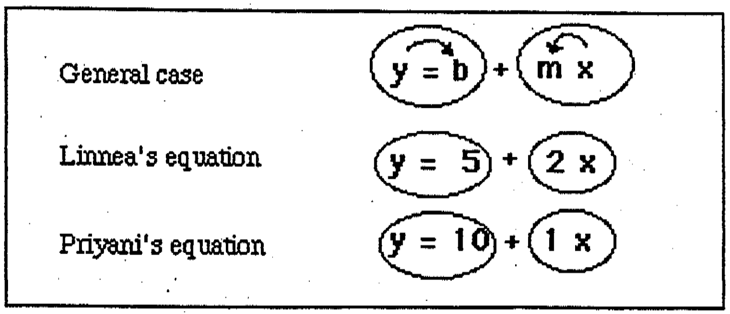

Figure 3. How some students appeared to parse a linear equation.

For example, Linnea explained how she created her equation: "This is the y-axis and this is the x-axis, and so I did a y equation, and so it's going up by 5, the y, and the x is going up by 2." This suggests that Linnea read $y = 5 + 2x$ as follows: "$y = 5$", which she interpreted as "the ys (on the y-axis) go up by 5"; "+", which meant "and"; and "$2x$", which meant "the xs (on the x-axis) go up by 2". Priyani spoke about her equation $y = 10 + 1x$ in a similar manner. She chose 10 as the "starting value" because her line of best fit for the data in Figure 2 intersected the y-axis at 10. She calculated the 1 in her equation as the change in y-values for two selected points, and she referred to the 1 as what the line goes up by. When the interviewer asked Priyani about the meaning of x and y in her equation of $y = 10 + 1x$, she explained that "y is the starting value" and "x is what it's going by".

Results of the Classroom Analysis

The Nature of the Teacher's Questions

We were surprised by the results of the interviews because the participants were the higher-performing and more diligent students in the classroom, and they had regularly explored linear relationships across a variety of real-world dependency situations. Consequently, we turned to an examination of the nature of the questions and commands that the teacher made during direct instruction and whole-class discussions. Our objective was to determine whether the teacher directed students to focus on the *relationship* between corresponding x- and y-values, or whether she primarily directed students to focus on *calculating* values for the y-intercept and slope and inserting them into the $y = \square \pm \square x$ equation. That is, we sought to identify any focusing phenomena involving the teachers' questioning patterns that might be related to whether or not the students attended to a functional relationship between corresponding x-and y-values.

The classroom analysis draws on the interpretive and iterative techniques used in the development of grounded theory (Glaser & Strauss, 1967; Strauss & Corbin, 1990). The two categories that emerged during our first round of analysis were *relational* utterances and *calculational* utterances. After further examining the videotaped data, it became apparent that the types of questions the teacher posed differed depending upon whether the classroom discussion centred on an informal discussion of real-world data, or whether the discussion included the use of the conventional representations of equations, tables, and graphs. Thus a second set of categories emerged, namely *situational* versus *representational*. By considering all combinations across both categories, four codes emerged: (a) situational relationship, (b) situational calculation, (c) representational relationship, and (d) representational calculation.

During the second round of analysis, we were unable to code a small percentage of the teacher's utterances using the four codes. As a result, a fifth code emerged—representational object—to account for the uncoded utterances. This code refers to a question or command uttered in a representational situation that treats a graph of a linear equation as an object or entity rather than as a representation of a relationship between x- and y-values. For example, the teacher might tell students that a line rising to the right has a positive slope. In this case, she is treating the sign of the slope as a characteristic of the line as an object and is not relating the sign to a relationship between x- and y-values, nor is she calculating a slope value. This fifth code allowed us to preserve the integrity of the data by accounting for all of the teacher's questions and commands. However, this code is not central to the main findings of this paper. The primary result of this analysis reveals a shift in calculational versus relational questions depending upon whether the context is situational or representational. Table 1 provides a description and example of each of the five codes.

Every instructional lesson involved both realistic situations and conventional representations. Thus situational and representational refer to what the teacher talked about rather than to the nature of the general activity in the classroom. That is, representational does not mean context-free, since realistic situations were discussed during every lesson; representational means that the teacher referred to conventional representations in the realistic situation. Suppose the teacher pointed to an equation or a graph representing data about the enlargement factor of an overhead projector and asked, "What do you notice is happening to the enlargement factor every time you increase x by 2 metres?" We would code the utterance as a representational relationship rather than situational relationship because she specifically directed students' attention to a conventional representation. Situational codes were used when the teacher referred only to the relationships or quantities in a situation, and a conventional representation had not yet been generated.

The codes refer to the teacher's questions in the form of inquiries or commands uttered during whole-class discussion or during direct instruction. We did not code the teacher's utterances as she walked around the room and informally talked to various small groups or individual students. The codes include every question the instructor uttered, including repeated questions. When the nature of the question was unclear, codes were assigned based upon the type of answer that the instructor accepted as appropriate. The totals for each type of coded utterance are shown in Table 2.

Of the 984 questions coded, only 156 (or about 16%) were relational in nature. A different picture emerges, however, if one examines the questions the instructor asked within situational contexts versus those asked within representational contexts. Seventy-nine of the 138 situational questions (57%) emphasised the relationship between x and y. In contrast, only 77 of the 846 questions (9%) in the representational context emphasised the relationship between x and y. The remaining 91% of the instructor's questions focused on finding slope and intercept values for $y = \square \pm \square x$ or treated the line as an object. The first 10 days of instruction were coded rather than the entire instructional unit, since the interviews began after Day 10. We did not have videotaped data for Day 9 due to technical difficulties. However, analysis of two sets of detailed field notes indicates a similar pattern for Day 9 as for Day 10, with approximately 90% of the content for Day 9 coded as representational calculation (RC) and 10% as representational object (RO).

Table 1
Codes for the Types of Questions or Commands Uttered by the Teacher about Linear Relationships During Formal Instruction

Code	Meaning	Example
SR	Situational Relationship. A question or command uttered during a discussion of a real–world situation that highlights the relationship between quantities in the situation.	Day 1, 24:27: "How many times bigger do you think the projector is making the image?" [Note: Teacher directs students' attention to the relationship between the length of an image and its projected image.]
SC	Situational Calculation. A question or command uttered during a discussion of a real-world situation that leads the student through a calculation that is not connected to the relationship between quantities.	Day 1, 1:39:00: "If we don't want to do this by guessing and checking with multiplication, what's another way we can use the calculator to get this more quickly? What would you divide?"
RR	Representational Relationship. A question or command uttered during a discussion of a symbolic representation that focuses on the relationship between x- and y-values.	Day 5, 1:40:00: "What happens to y as x grows larger (referring to a linear equation)? Is it getting bigger or smaller?"
RC	Representational Calculation. A question or command uttered during a discussion of a symbolic representation that focuses on the procedures for either determining values for the missing values in $y = \square \pm \square x$ or on whether to use + or –, but does not focus on the relationship between x and y.	Day 8, 1:15:00: "What number do I put into the first box (referring to $y = \square \pm \square x$ on the overhead)? What is the starting value?"
RO	Representational Object. A question or command that focuses attention on the graph of a linear equation as an object rather than as a representation of a relationship between x and y.	Day 2, 1:06:00: "How do we decide where we draw that linear model (referring to a given scatterplot)? We have to place it so that half of the points are above and half are below the line."

Table 2
Number of Each Type of Coded Utterance Made During the First Nine Days of the Instructional Unit

	Situational Context			Representational Context				
	SR	SC	$Total_S$	RR	RC	RO	$Total_R$	Total
Day 1	16	35	51	0	0	8	8	59
Day 2	24	14	38	19	55	9	83	121
Day 3	1	1	2	18	40	40	98	100
Day 4	0	9	9	0	51	56	107	116
Day 5	3	0	3	14	62	2	78	81
Day 6	0	0	0	2	27	14	43	43
Day 7	28	0	28	6	144	4	154	182
Day 8	4	0	4	17	192	18	227	231
Day 10	3	0	3	1	43	4	48	51
Total	79	59	138	77	614	155	846	984
% of 984	8	6	14	8	62	16	86	
% of $Total_S$ or $Total_R$	57	43		9	73	18		

Note. SR = situational relationship; SC = situational calculation; RR = representational relationship; RC = representational calculation; RO = representational object; $Total_S$ = Total for situational context; $Total_R$ = Total for representational context

When the instructor engaged in discussions about the real-world phenomena to be modelled, a majority of her questions were relational in nature. She emphasised the relationship between the dependent and independent variables as students explored linearly-changing quantities. However, as soon as conventional representations of the phenomena were generated, a shift in focus occurred. The number of questions emphasising a relationship between x and y decreased dramatically, and the instructor instead highlighted methods for calculating the values to place in the boxes in the equation $y = \square \pm \square x$. Consequently, it is not difficult to see why students interpreted $y = \square \pm \square x$ as a storage container, since the vast amount of attention directed to this representational form was calculational in nature. In fact, much of the whole class instruction regarding equations was organised around the calculation of values to place in the boxes in $y = \square \pm \square x$.

We present two illustrative classroom examples from the first activity in the unit. The first example demonstrates a typical discussion in which the teacher focused on the relationship between independent and dependent variables in the overhead projector situation. The second example shows how the discussion shifted to a calculational focus once the representation of an equation was introduced. The description of this introduction of linear equations will illustrate how the instruction supported the development of the conception of a linear

equation as "y = 'starting point' plus 'what is goes up by' x". Further, this conception of an equation discouraged a focus on any dependency relationship between x and y, and instead supported a view of the equation as a storage container.

Example of a Relational Discussion

When the instructor engaged in discussions about the phenomena to be modelled prior to the introduction of conventional representations, the majority of her questions emphasised the relationship between the dependent and independent variables. For example, the CPMP unit opened with a 3-day activity in which students explored a linear relationship between the distance of an overhead projector from the wall and the resulting enlargement factor between the length of an object and its projected image. The teacher and the students initially explored the phenomenon of an enlargement factor and discussed the dependency relationship between the length of an object and its projected image:

Ms R: So what's going to happen to the size of this shape if I put it on this projector?
Students: It's going to get bigger.
Ms R: It's going to get bigger? So, this is the measurement I have on the paper. [She shows the class with her fingers how big her drawing is.] This little tiny thing. Right? If I go up here, look how much bigger it is. How many times bigger do you think this projector is making that image?
Marc: Two.
Ms R: Two times bigger?
Marc. Three.
Ms R: Three times bigger maybe? So that's what we're going to find out. How many times bigger is this projector enlarging it.

Thus the teacher directed students' attention to the multiplicative relationship between the size of an object and its projected image. She posed a thought experiment for the students to consider, in which the length of an object was 2 cm and its projected image was 16 cm. Several students said that the enlargement factor would be 8 in that case. The teacher emphasised the multiplicative relationship by focusing on the fact that an enlargement factor of 8 meant the image was 8 times larger than the original object. She continued to encourage students to consider dependency relationships as they switched to consider a new situation involving the relationship between the distance from the projector to the wall and the corresponding enlargement factor:

Ms R: Now do think that will happen if I move the projector closer? Will the enlargement factor change if I put it closer and I focus it?
Student: It will change.
Ms R: Which way do you think it will go?
Student: Down.
Ms R: Down? So the enlargement factor will be smaller or bigger? Smaller. Okay. That's what we're going to look at right now. Our first experiment in this investigation is we're going to try to figure out the enlargement factor of this projector. But the enlargement factor of the projector depends on something. What does it depend on? How far away it is? What size you're starting with? So those are all the questions we're going to look at.

The teacher focused the students' attention towards dependency relationships by asking students questions about the relationship between independent and dependent variables. However, once the students created a table and had to write an equation, a shift in focus occurred. The class collected data in the following manner. The teacher placed the projector 2 m from the board. Then she called on students to collect data for the lengths of the projected image for each of four sketch lengths: 10 cm, 27 cm, 5 cm and 15 cm. They computed an average enlargement factor from these four data points. The students entered the enlargement factor of 4.8 in a table. The teacher then moved the projector back 1 m until it was 3 m from the board, and the data collection process was repeated. Once the first three rows of data were collected and compiled in the table shown in Table 3, the nature of the teacher's questions shifted in focus, as will be demonstrated in the next section.

Table 3
Data collected by students showing the enlargement factor of an overhead projector placed varying distances from the board

Distance from Board	Enlargement Factor
2	4.8
3	7.8
4	10.3

Example of a Calculational Discussion

The teacher invited the students to look for patterns in the table of data shown in Table 3:

Ms R:	What does that 10.3 tell us then?
Student:	It goes up by 3.
Ms R:	It goes up by 3? So you mean this way it goes up by 3 [sweeps hand vertically down the column of y-values in the table]?
Student:	Yeah.
Ms R:	That looks like a good pattern. Approximately 3? Very good.

The teacher's gesture of sweeping vertically down the y-column likely focused attention on the iterative pattern in the y-values. In addition, the "goes up by" language also contributed to a focus on the y-values changing rather than on a coordination between x-and y-values. When asked to predict the enlargement factor when the projector was 5 m and 6 m from the wall, several students responded that the enlargement factor would be 13 and 16, respectively, suggesting that they attended only to the difference between successive y-values. After additional data were collected (5 m, 13.0; 6 m, 15.8; and 1 m, 2.2), the teacher again invited the students to look for patterns in the table:

Ms R:	Let's see. Sandra, can you tell us what pattern you see in the table?
Sandra:	It goes up by like 2 or 3.
Ms R:	Okay. So she says that it increases by about 2 or 3. Every how often does it increase by 2 or 3?
Sandra:	Every metre.
Ms R.:	Every metre. Okay. That's good.

The teacher likely understood "about 2 or 3" as a ratio since she prompted Sandra to notice how much the enlargement factor increased for each metre. However, that value could have simply represented a difference between y-values for Sandra and others. The instructor then explicitly directed the students to focus on the iterative relationship among y-values by asking, "So if we took one of the measurements that we have right now, how would we get to the next number; would you add exactly 2 or exactly 3?" In response, the students suggested finding the average of the differences between successive y-values, which was 2.7. The teacher then emphasised that 2.7 meant "to get to the next number, you add approximately 2.7, so 2.7 is how much you do to one number to get the next number". Thus she encouraged the students to focus on the recursive relationship between successive y-values rather than on the dependency relationship between corresponding x- and y-values.

The teacher then linked 2.7 to the equation $y = \square \pm \square x$ with language that supported the notion that the students must add 2.7 "every time". She explained that students should first write the starting number, which is the "starting number at the very beginning of the line". She then instructed students to write "+ 2.7x" because they added 2.7 every metre:

Ms R:	Two point seven [writes 2.7 to show $E = 0 + 2.7$]. And how often are we adding 2.7?
Students:	Every 1 metre.
Ms R:	Every metre. So what do you want to put after the 2.7?
Students:	x.
Ms R:	x [adds x to the equation so it reads $E = 0 + 2.7x$]. Can we put any letter we want?
Students:	Yeah.
Ms R:	Sure. If you want, put x to stand for the distance or d for the distance, whatever letter you want to use.

The teacher likely understood that the students should write 2.7x because the enlargement factor increased by 2.7 for every 1 metre. However, because the students attended to the recursive relationship among the y-values rather than to a multiplicative comparison between x and y, they might have interpreted the teacher's directions to mean that one should add an x to the equation because they added 2.7 every metre (or "every time").

In addition, the instructor focused the students' attention on determining 2.7 as what one would add to get to the next number independent of any consideration of y or x. The only discussion of x occurred when the students had to write the $y = \square \pm \square x$ equation. At that point the instructor directed the students to add x to the 2.7 in the equation, and then continued by stating that any letter could stand for the distance. She referred to x or d as "stand(ing) for the distance", rather than emphasising that x represents a numeric value for a given distance. The teacher and the students also never discussed the meaning of y in $y = \square \pm \square x$. Ms R no longer posed questions emphasising a relationship between x and y, as she had when the projector situation was first introduced. Instead, she emphasised a calculational strategy for writing an equation by first writing a mathematical sentence with empty boxes and then showing the students how to determine the values of the boxes.

It is not difficult to see the origins of the students' interpretation of a linear equation as "y equals the 'starting value' plus 'what it goes up by' x" in light of the instructional treatment described above. Although the teacher may have used language and particular questions that were unintended by the CPMP authors, it is important to note that the sequence of observed activities in the classroom for the overhead projector investigation followed the sequence identified in the CPMP text. The teacher's method for writing an equation described above evolved into the standard mathematical practice for determining the equation of a line. Each time the students were confronted with linear data, the teacher instructed them to locate the starting value, determine "what it goes up by", and then to enter these two values into the boxes in $y = \square \pm \square x$. Throughout the remainder of the instructional unit, the students and the teacher explored dependency relationships in a variety of situations, including ticket prices and concert attendance, the stretch of rubber bands and springs, the cost for soda machines, and speed with motion detectors. However, once conventional representations were introduced, the instruction in each case inevitably shifted from a relational to a calculational focus.

Discussion

The results of this study provide one plausible explanation for students' interpretation of a linear equation as a storage container, given the nature of the instructional environment. The students' interpretations appeared to be linked to the focusing phenomena of the teacher's questioning pattern, which frequently and regularly directed students' attention to the calculation of b and m values and the insertion of these values into the boxes in $y = \square \pm \square x$. We conjecture that by altering the nature of the focus of attention, we can significantly affect the nature of students' interpretations of linear equations. If instruction regularly directs students' attention to the relationship of covarying quantities, then students are more likely to generate meaning for a linear equation as a statement about the relationship between corresponding x- and y-values.

This study is limited in what it reveals about individual students' understanding of dependency relationships in realistic situations. It was not designed to determine whether the primary source of difficulty rests with students' inability to form a linear relationship in realistic situations, or with their inability to connect the realistic situation to conventional symbols. Most likely, many students experienced both types of difficulties. Consequently, we believe that a three-pronged approach is indicated in order to support students' connections between realistic situations and conventional symbol systems. Specifically, there is evidence that instruction should direct students' attention to relationships between covarying quantities in each of three places: (a) in the realistic situations, (b) when equations are created to represent relationships among quantities in situations, and (c) when equations are interpreted or "mapped back" into the real-world situation. We will consider each instructional idea in terms of Ms. R's classroom and the overhead projector situation.

Idea 1

The first idea is to continue posing relational questions in realistic situations until students have formed a linear relationship between quantities in the situation, and have conceived of the rate of change of the function as a conceptual entity. For example, it is unclear whether the students in this class correctly interpreted the 2.7 in $y = 2.7x$ as the *change* in enlargement factor or thought that 2.7 was simply the enlargement factor. The discussion of the overhead projector situation began with a consideration of the relationship between the length of an object and its projected image. The conversation moved quickly to the consideration of a second function, namely the relationship between the distance of the projector from a wall and the enlargement factor. It is doubtful that the constancy of the enlargement factor was established for the first relationship before moving to the second relationship.

The teacher seemed to effectively direct students' attention to the multiplicative relationship between objects and their projected images. However, once the class determined that the 16 cm projected image was 8 times as large as the 2 cm object, they moved on. The students missed the opportunity to consider whether this multiplicative relationship was constant when the length of the object varied. In contrast, a teacher could ask students what they think will happen when she places an object that is 12 cm long on the projector? Similarly, students could consider the size of the enlargement factor for objects with a range of lengths. Once the constancy of the enlargement factor is established for the projector at a fixed distance from the wall, it makes more sense to consider what happens when the projector is moved.

Prior to this study, we thought that students would form linear relationships between independent and dependent variables because of their extensive hands-on experience with those relationships. For example, because of physically moving an overhead projector away from a wall one metre at a time and calculating the subsequent enlargement factor, we thought that students would likely notice a relationship between distance and the change of the enlargement factor. We now believe that the dependency nature of linear relationships is not inherent in the situations but requires focus and attention. We need to find ways to redirect students' attention away from the recursive relationship between successive enlargement factor values and toward the relationship between the distance and the enlargement factor. For example, different groups of students could collect data for different values of the distance from the screen, yielding different enlargement factors, and then share these findings in a common format and consolidate them in order to identify a common underlying relationship.

Idea 2

The results of the study suggest that it is important for the teacher to continue to pose relational questions during the generation of the equation. One could argue that the students learned exactly what they were taught, namely that a linear equation is "y equals 'the starting point' plus 'what it goes up by' x". However, that is not quite accurate. The results of the analysis of the classroom discussion in which $y = 2.7x$ was generated suggest that while the teacher was able to interpret the equation as a dependency relationship, many of the students were not. Ms R indicated that she interpreted 2.7 as the change in the enlargement factor for each 1 m of distance between the wall and the projector. She likely interpreted $2.7x$ as the enlargement factor for a particular distance x and viewed $y = 2.7x$ as a relationship between corresponding distances and enlargement factors. In contrast, the students appeared to have focused their attention on 2.7 as part of a recursive numeric pattern. Thus, it is unclear what 2.7 meant for them in terms of the quantities in the situation. Furthermore, x was likely interpreted as a label meaning "every time". The ambiguity of terms such as "goes up by" and "starting value" allowed for multiple interpretations of $y = 2.7x$. A successful alternative instructional approach is one that better aligns the teacher's interpretation of the *product* of the discussion, namely, the equation $y = 2.7x$, with the *process* by which she helps the students generate the equation.

Idea 3

Once the formal symbolic representation of equations has been introduced, this study indicates that it could be critical to ask students to interpret the meaning of each symbol and each arithmetic operation in terms of the referents in the realistic situation. For example, the classroom discussion surrounding the generation of the equation $y = 2.7x$ would have looked quite different had the teacher posed questions such as the following:

- What do the x and the y represent in this overhead project situation?
- What does the 2.7 mean? Is it a distance, an enlargement factor, a projected length or something else?
- What quantity is represented by $2.7x$?

The language of quantities and units can focus attention on the development of the dependency relationship in the situation and on connections between meaning and symbols. Thompson (1995) recommends that teachers should insist that any time students speak of a number within a real-world situation, they speak of the quantity for which the number is a value. Furthermore, when students use an arithmetic operation, they should name the quantities on which they are operating, the quantity their calculation evaluates, and what the operation accomplishes in the situation.

The ideas suggested by this study are in keeping with the spirit of Sfard's (2000) call "to orchestrate and facilitate the back-and-forth movement between symbols and meanings" (p. 92). We believe that work needs to be done on three fronts: in developing meaning for each referent as a conceptual entity; in using symbols to meaningfully represent the dependency relationships in situations; and in continuing to direct attention back to dependency relationships once the conventional symbols are introduced. Further research needs to determine what teaching strategies are likely to be successful in these educational tasks.

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Blocking the Growth of Mathematical Understanding: A Challenge for Teaching

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This paper presents and discusses some of the findings of a research project that focused on teaching and learning in two high-school mathematics classrooms. The focus of the study was to consider the ways in which teachers' classroom interventions promote the growth of students' mathematical understanding. Analysis of the data resulted in the generation of a number of themes describing the teachers' interventions. One of these themes, that I call *blocking*, is the subject of this paper. The paper discusses the implications of this intervention strategy for teaching, learning, research, and teacher education.

In 1990, Stevens, reporting on the deliberations of an International Congress on Mathematical Education discussion group of which he had been a member in 1988, touched upon the problem of "when it is appropriate for a teacher to intervene in order to redirect a child's thinking" (p. 231). Stevens reported that the group was unable to reach agreement, and suggested that the issue was unresolved. The dilemma of when to intervene is an enduring one for teaching, and continues to perplex and challenge practitioners and researchers alike. This paper addresses the dilemma of when to intervene by presenting episodes of classroom data that demonstrate how particular teaching strategies and interventions may in fact inhibit rather than promote the growth of students' mathematical understanding.

There exists a comprehensive body of research on teachers' behaviour, talk, and interaction in classrooms, much of which, in one way or another, addresses this dilemma. In particular, such areas as the nature of teachers' talk in mathematics classrooms, and particularly their use of questions (Martino & Maher, 1994; Vacc, 1993), have been widely discussed. For many years a great variety of authors have been pointing to the preponderance of teachers' questions in the classroom while bemoaning the dearth of student ones (Chuska, 1995; Dillon, 1987, 1988; Postman & Weingartner, 1969). Research has shown that teachers commonly ask as many as 50,000 questions a year and their students as few as 10 each (Watson & Young, 1986). There are powerful reasons why teachers ask questions. Edwards (1980) points out that like lawyers, doctors, and police officers, teachers learn to ask questions, indeed may have been trained to ask questions, that restrict the scope of the answers, so as to get only as much as is required. (What is required is open to debate, of course, but in this sense it is taken to mean required-in-order-to-keep-the-locus-of-control-firmly-in-the-hands-of-the-teacher.) Edwards and Mercer (1987) report that interviewers, therapists, barristers, and others whose job it is to ask questions are typically advised that asking strings of direct questions is the surest way of shutting up the interviewee. It should come as no surprise to us, then, that for many teachers one function of questioning is to maintain control of the classroom, so that intervening is a management strategy as much as a teaching strategy.